Uninvadable social behaviors and preferences in group-structured populations

Jörgen Weibull

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This talk is based on joint work with

- *Ingela Alger* (Toulouse School of Economics, and Institute for Advanced Study)

- *Laurent Lehmann* (Lausanne University, Department of Biology)

We have so far written two manuscripts:

- Lehmann, Alger & Weibull (2015): "Does evolution lead to maximizing behavior?", *Evolution*

- Alger, Lehmann & Weibull (2018): "Uninvadable social behaviors and preferences in group-structured populations", WP

1 Questions

Imagine

1. A large population divided into (a large number of) identical locations, or *islands*, with n individuals on each island. Each individual occupies one *slot*

2. The n individuals on an island interact with each other. The interaction is represented as a normal-form game

3. The set of strategies available to each player is a non-empty and compact set X in some normed vector space

4. Payoff functions are continuous, and *aggregative symmetric* in the sense that a player i's payoff is independent of player identity i, and is permutation invariant w.r.t. other player's strategies, $\vec{x}_{-i} \in X^{n-1}$. Write $\pi(x_i, \vec{x}_{-i})$ for this payoff, and F for the function class

5. There is a type space Θ, and each individual's type is her *private information*. We analyze two cases:

   (a) A type is a strategy: $\Theta = X$

   (b) A type is a goal function: $\Theta = F$ (for example altruistic or spiteful, inequity averse)

6. In the evolutionary selection processes to be studied, types *breed true*; offspring are *clones* of their (single) parent

7. Evolution (natural or cultural) takes place perpetually over discrete time, as follows:

   (a) In each time period, the game is played, and thereafter some individuals get offspring, some survive and others die. Those with higher payoffs get more offspring and/or are more likely to survive.

   (b) Offspring compete for empty slots (that arise when an "old" individual dies). They either compete in their native island, or go to another island and compete there. The latter are *migrants*.

   (c) Migration decisions are i.i.d. The probability for migrating is fixed, $m \in (0, 1)$, and those who migrate choose a destination island (uniformly) randomly

   (d) Empty slots (if any) are filled by a uniform random draw among candidates (local and migrant offspring)

   (e) Offspring that do not obtain a slot die

8. Suppose that initially all individuals in the whole population is of the same type $\theta \in \Theta$, and that we replace one of these individuals, on one island, by a *mutant*, an individual of another type $\tau \in \Theta$. The first, incumbent type, θ, is called *uninvadable* if the lineage of every such mutant $\tau \neq \theta$ gets extinct in finite time with probability one

9. Our questions:

   (a) Does there exist any uninvadable type?

   (b) How do uninvadable types behave? What is the nature of their goal functions?

   [Are they *Homo oeconomicus*, that is, rational and selfish in the sense of striving for maximal payoff, playing best replies against each other? Do they use evolutionarily stable strategies? Or are they altruistic or spiteful? Can they have any morality?]

10. We cannot answer these questions as stated. However, we can answer them under so-called *weak selection*, that is when the effects of behaviors or goal functions, via payoffs, on fitness, are small.

2 Definitions and preliminaries

- Consider any game $G = \langle n, X, \pi \rangle$ with properties as described above

- Let $\Theta = X$

2.1 Individual fitness

- 1. An individual's *fitness* is defined as the expected number of *immediate descendants*; her surviving offspring and herself if she survives

  2. This fitness in general depends on (a) the individual's own payoff, (b) the payoffs to the other $n-1$ individuals on her island, and (c) the average payoff in the population at large. Write $w(\pi_i, \vec{\pi}_{-i}, \pi^*) \in \mathbb{R}_+$

3. We assume that individual fitness is invariant under permutation of the payoffs to the $n-1$ island neighbors.

4. We also assume the following monotonicity properties:

[M] *(i) Fitness is strictly increasing in own material payoff. (ii) The marginal effect of a neighbor's material payoff on fitness is not larger than the marginal effect of own material payoff. (iii) Fitness is decreasing in the average material payoff in the population at large.*

2.2 Lineage fitness

- An individual's *lineage* is defined as the collection of this individual's all descendants; her immediate descendants, the immediate descendants of her immediate descendants, etc. *ad infinitum*

- Suppose that initially everybody in the population is of the same type $\theta = x \in X$, and that suddenly one individual on an island, a *mutant*, switches to another type, strategy $y \in X$.

- The mutant's *local lineage* is defined as the collection of all descendants of the mutant who live or lived on the individual's native island

- The random time, T, of the first extinction of the mutant's local lineage is finite with probability one, since the migration probability, m, is positive and constant

- The *lineage fitness* of the mutant strategy y is defined as the average fitness of a mutant's local lineage members. Let $\pi(y|k)$ and $\pi(x|k)$ denote the material payoffs to individuals using strategies y and x, respectively, in any island in which exactly $k+1$ individuals use strategy y and the others use x. The lineage fitness of strategy y can then be written in the form

$$W(y,x) = \sum_{k=0}^{n-1} p_k(y,x) \cdot w\left(\pi(y|k), \langle \pi(y|k), \pi(x|k) \rangle, \pi^*(x)\right).$$

- Here $p_k(y,x)$, for $k = 0, 1, ..., n-1$ is the probability of a (uniformly) randomly drawn lineage member (a mutant), observed in any time period $t = 0, 1, ..., T$, to coexist with k other lineage members (mutants)

3 Answers

Proposition 3.1 (Lehmann, Alger & Weibull, 2015; Lehmann et al., 2016) *A strategy* x *is* uninvadable *iff*

$$W(y, x) \leq 1 \quad \forall y \in X.$$

- Unity is the lineage fitness of a pseudo mutant (one who behaves exactly as the incumbents); $W(x, x) = 1 \ \forall x \in X$

- Equivalently, the characterizing criterion can be written as

$$x \in \arg\max_{y \in X} W(y, x).$$

- A strategy is thus uninvadable iff it is a best reply to itself in terms of lineage fitness

- This provides the first step in the search for answers to our questions

- In order to take the next step towards answering our questions, we consider weak selection

3.1 Weak selection

- In biology "weak selection" refers to situations in which the fitness effects of heritable traits are small

- Formally, we assume that an individual's payoff affects her survival probability and expected number of offspring by multiplicative factors of the form $e^{\delta \pi_i}$ for $\delta > 0$, and study the limit as $\delta \to 0$

- The key implication of this is that, as $\delta \to 0$:

$$p_k(y,x) \to p_k(x,x) = p_k^0 \qquad \forall x, y \in X,\ k = 0, 1, ..., n-1$$

- Let $\boldsymbol{p}^0 = \left(p_0^0, p_1^0, ..., p_{n-1}^0\right)$

- Define the mutant strategy's *lineage payoff-advantage* as

$$\Pi(y,x) = \sum_{k=0}^{n-1} p_k^0 \cdot \tilde{\pi}^{(k)}(y,x),$$

where

$$\tilde{\pi}^{(k)}(y,x) = \pi(y|k) - \lambda_0 \cdot \left[\frac{k}{n-1}\pi(y|k) + \frac{n-1-k}{n-1}\pi(x|k)\right].$$

- The parameter λ_0 depends on population structure and the evolutionary transmission scenario, but it does not depend on the material payoff function π *per se*. We refer to λ_0 as *local competitiveness*

Proposition 3.2 *Under weak selection, a strategy $x \in X$ is uninvadable iff*

$$\Pi(y,x) \leq \Pi(x,x) \quad \forall y \in X.$$

- Equivalently, a strategy is a uninvadable if it is a best reply to itself in terms of lineage payoff advantage. Preemption.

3.2 Nash equilibrium

- Consider the goal function u^0, defined by

$$u^0(x_i, \boldsymbol{x}_{-i}) = \mathbb{E}_{\boldsymbol{p}^0}\left[\pi(x_i, \tilde{\boldsymbol{x}}_{-i}) - \lambda_0 \cdot \sum_{j \neq i} \pi\left(\tilde{x}_j,\ \tilde{\boldsymbol{x}}_{-j}\right) \mid \boldsymbol{x}\right] \quad \forall \boldsymbol{x} \in X^n$$

  - We call such individuals *competitive moralists*.

Corollary 3.3 *A strategy $\hat{x} \in X$ is uninvadable under weak selection if and only if it is a symmetric Nash equilibrium strategy in the n-player game in which every player's strategy set is X and every player has payoff function u^0.*

3.3 First-order condition for uninvadability

- The *pairwise relatedness coefficient*

$$r_0 = \sum_{k=0}^{n-1} \frac{k}{n-1} p_k^0,$$

is the expected share of other lineage members among one's $n-1$ neighbors

Corollary 3.4 *Let $\pi : X^n \to \mathbb{R}$ be continuously differentiable with $X \subseteq \mathbb{R}$. If a strategy $\hat{x} \in int\,(X)$ is uninvadable under weak selection, then*

$$\left[\pi_1 (y, \hat{\mathbf{x}}) \; + \; (n-1)\, \kappa_0 \cdot \pi_n (y, \mathbf{y})\right]_{y=\hat{x}} = 0,$$

where $\hat{\mathbf{x}}$ is the $(n-1)$-dimensional vector whose components all are $\hat{x}$, and $\mathbf{y}$ is the $(n-1)$-dimensional vector whose components all are y, and

$$\kappa_0 \;= \frac{1}{1-\lambda_0 r_0}\left[r_0 - \lambda_0 \cdot \frac{1+(n-2)r_0}{n-1}\right].$$

4 Preference evolution

- Let $\Theta = F$

- Evaluate payoffs in all (Bayesian) Nash equilibria under incomplete information (where an individual's type is her private information)

- For any goal function u, let $X(u)$ be its set of symmetric Nash equilibrium strategies,

$$X(u) = \left\{ \hat{x} \in X : \hat{x} \in \arg\max_{x \in X} u\left(x, \hat{\mathbf{x}}^{(n-1)}\right) \right\}.$$

- Our main result for preference evolution:

Theorem 4.1 *The utility function u^0 is uninvadable under weak selection. A utility function $u \in F$ is invadable under weak selection if there exists a $\hat{x} \in X(u)$ such that $\hat{x} \notin X\left(u^0\right)$.*

5 Examples

5.1 Payoff-dependent fecundity

- Assume constant survival probability, s_0, and Poisson-distributed number of offspring, with mean value $f(\pi_i)$. Then

$$w(\pi_i, \boldsymbol{\pi}_{-i}, \pi^*) = s_0 + (1 - s_0)\, m \cdot \frac{f(\pi_i)}{f(\pi^*)} + (1 - s_0)(1 - m) \cdot \frac{f(\pi_i)}{(1 - m)\, f(\bar{\pi}) + nmf(\pi^*)}$$

- Assume

$$f(\pi_i) = e^{\delta \pi_i} \text{ for some } \delta > 0.$$

Then pairwise relatedness becomes

$$r_0 = \frac{(1-m)^2 + \left(1+m^2\right) s_0}{n - (n-1)(1-m)^2 + \left(1-(n-1)m^2\right) s_0}$$

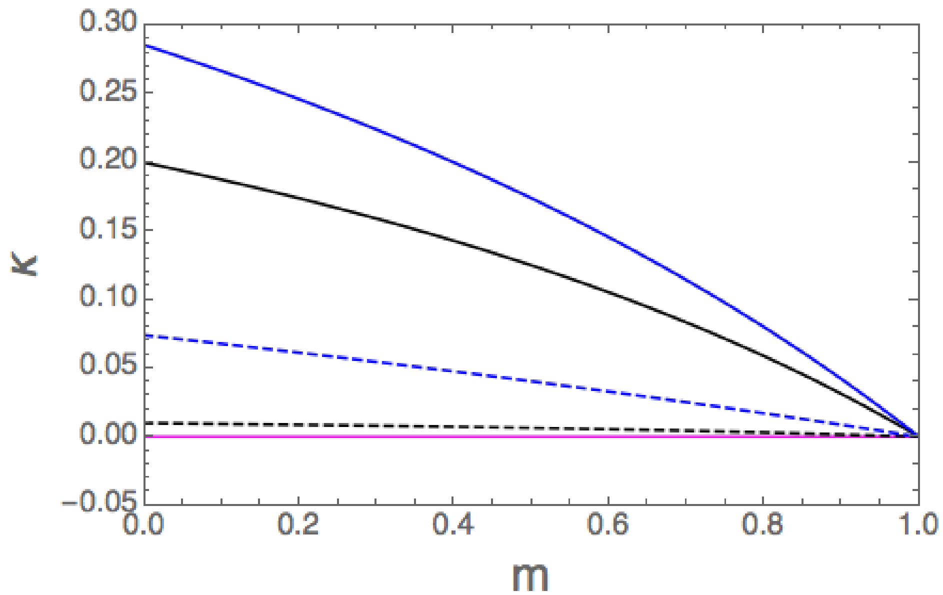
and local competitiveness is

$$\lambda_0 = \frac{(n-1)(1-m)^2}{n-(1-m)^2}.$$

- Hence:

$$\kappa_0 = \frac{2(1-m)s_0}{2(1-m)s_0 + n\left[2 - m(1-s_0)\right]}$$

- The associated goal function u^0 has three (additive) components: (a) self-interest, (b) spite (envy), and (c) a form of Kantian morality (act in a way that you would like others on your island to act)



5.2 Payoff-dependent fecundity and winning-probability in wars

- Assume zero survival probability: $s_0 = 0$, probability $\rho \in (0,1)$ for war, and conditional winning probability

$$v_n(\bar{\pi}, \pi^*) = \frac{\exp(\delta n \bar{\pi})}{\exp(\delta n \bar{\pi}) + \exp(\delta n \pi^*)}.$$

- Pairwise relatedness is unaffected by the risk of war (when $s_0 = 0$):

$$r_0 = \frac{(1-m)^2}{n-(n-1)(1-m)^2}$$

but local competitiveness is (monotonically) affected:

$$\lambda_0 = \frac{2(n-1)(1-m)^2 - \rho(n-1)n}{(2+\rho)n - 2(1-m)^2}.$$

- Hence:

$$\kappa_0 = \frac{\rho}{\rho + 2m\,(2-m)}$$

- The goal function u^0 still has three (additive) components: (a) self-interest, (b) net spite, and (c) a form of Kantian morality, where net spite changes sign at a critical value of $\rho \in (0,1)$; The risk of war then turns spite/envy into altruism

6 Conclusion

Under weak selection:

1. Uninvadable strategies exist iff there exists a strategy that is a best reply to itself in terms of lineage payoff advantage.

2. Uninvadable strategies can be characterized in terms of pairwise relatedness and local competitiveness.

3. Uninvadable goal functions contain (additive) components of self-interest, morality, and spite/altruism.