A Stackelberg duopoly model with sticky prices and myopic follower

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GDO2018
March 13th, 2018
University of Vienna, Austria

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If we treat them as a Stackelberg duopoly, with player 1 being the Leader, we obtain:

- $q_1^{SB} = \frac{2(A-c)}{7}$,
- $q_2^{SB} = \frac{5(A-c)}{21}$ and
- $p^{SB} = \frac{10A+11c}{21}$.

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Extract from literature

[1] R. Cellini, L. Lambertini, 2004, *Dynamic Oligopoly with Sticky Prices: Closed-Loop, Feedback and Open-Loop Solutions*, Journal of Dynamical and Control Systems **10**, 303-314.

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[5] A.Wiszniewska-Matyszkiel, M. Bodnar, F. Mirota, 2015, *Dynamic Oligopoly with Sticky Prices: Off-Steady-state Analysis*, Dynamic Games and Applications **5**, 568-598, DOI 10.1007/s13235-014-0125-z.

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  - In our case, equivalent results, resulting in the same price trajectory.

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*Under some unpleasant technical assumptions A1–A3 the core relations (**CR**) of the Pontriagin maximum principle hold together with the terminal condition (**TC**).*

(A1) For almost all $t \geq 0$ and every $(x, u) \in \mathbb{X} \times \mathbb{U}$, partial derivatives $f_x(t, x, u)$ and $g_x(t, x, u)$ exist. The functions f and g and their partial derivatives with respect to x are Lebesgue-Borel measurable in (t, u) for every x, continuous in x for a.e. $t \geq 0$ and every fixed $u \in \mathbb{U}$ uniformly bounded as functions of t over every bounded set of (x, u).

(A2) There exist a continuous function $\gamma : [0, \infty) \to \mathbb{R}_+$ and a locally integrable function $\phi : \mathbb{R}_+ \to \mathbb{R}$ such that $\{x : \|x - x^*(t)\| \leq \gamma(t)\} \subseteq \mathbb{X}$ for all $t \geq 0$, and for a.e. $t \geq 0$, $\max_{x:\|x - x^*(t)\| \leq \gamma(t)} \left\{\|f_x(t, x, u^*(t))\| + \|g_x(t, x, u^*(t))\|e^{-rt}\right\} \leq \phi(t)$.

(A3) There exist a number $\beta > 0$ and an integrable function $\mu : \mathbb{R}_+ \to \mathbb{R}_+$ such that for every $\zeta \in \mathbb{X}$ with $\|\zeta - x_0\| < \beta$ the evolution equation (8) with $u = u^*$ and the initial condition replaced by $x(0) = \zeta$, has a solution on $\mathbb{R}_+$, denoted by x^ζ, and this solution fulfils for a.e. t, $\max_{x \in \mathrm{Conv}(x^\zeta(t), x^*(t))} \left| e^{-rt} \left\langle g_x(t, x, u), x^\zeta(t) - x^*(t) \right\rangle \right| \leq \|\zeta - x_0\|\mu(t)$.

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Open loop

Application of the Pontriagin maximum principle to optimization of player i, given the static best responce the follower.

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- The dynamics of the costate and state variables is given by

$$\dot{\lambda} = \begin{cases} (\frac{5}{3}s + r)\lambda - p + c & \text{for } (\lambda, p) \in \Omega_2, \\ (s + r)\lambda & \text{for } (\lambda, p) \in \Omega_1, \end{cases}$$

$$\dot{p} = \begin{cases} \frac{4}{9}s^2\lambda - \frac{5}{3}sp + s\frac{2A+3c}{3} & \text{for } (\lambda, p) \in \Omega_2, \\ -sp + s\frac{2A+c}{3} & \text{for } (\lambda, p) \in \Omega_1. \end{cases}$$

Costate-state dynamics for the optimal strategy

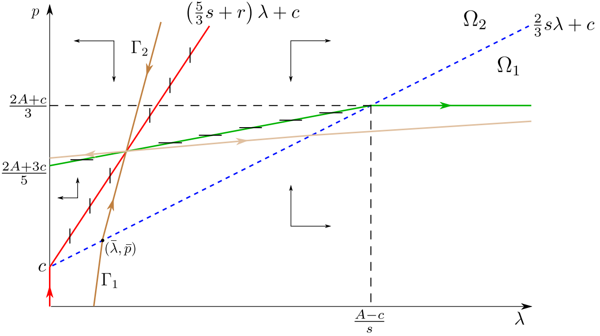

Figure: The phase portrait of the costate-state dynamics: Solid red line with vertical bars – λ-null-cline. Solid green line with horizontal bars – p-null-cline. Dark brown thick line with arrows denotes the stable saddle path. Dashed blue line is $p=\frac{2}{3}s\lambda+c$ that divides the first quarter into region Ω_1 (below this line) and Ω_2 (above it).

Open loop – results of the Pontriagin maximum principle

- The steady state of the costate-state dynamics is

$$\lambda^* = \frac{2(A-c)}{5r+7s}, \qquad \mathbf{p}^* = \frac{3r(2A+3c)+s(10A+11c)}{3(5r+7s)}.$$

- The corresponding steady state production of the leader is

$$\mathbf{q}_1^* = \frac{2(r+s)(A-c)}{5r+7s}.$$

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- Let us denote the intersection of the stable saddle path with the line $p = \frac{2}{3}s\lambda + c$ by $(\bar{\lambda}, \bar{p})$.

  If $p(t) < \bar{p}$ then $q_i(t) = 0$, otherwise $q_i(t) = p(t) - c - \frac{2}{3}\lambda(t)s$.

Optimal control in the feedback form – The Bellman equation

If a C^1 function V fulfils

- the Bellman equation $rV(p) = \sup_{q_1 \geq 0} pq_1 - cq_1 - \frac{q_1^2}{2} + V'(p)s(A - q_1(p) - q_2(q_1(p)))$

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- $q_1(p) \in \text{Argmax}_{q_1 \geq 0} pq_1 - cq_1 - \frac{q_1^2}{2} + V'(p)s(A - q_1(p) - q_2(q_1(p)))$ defines the optimal strategy of the Leader.

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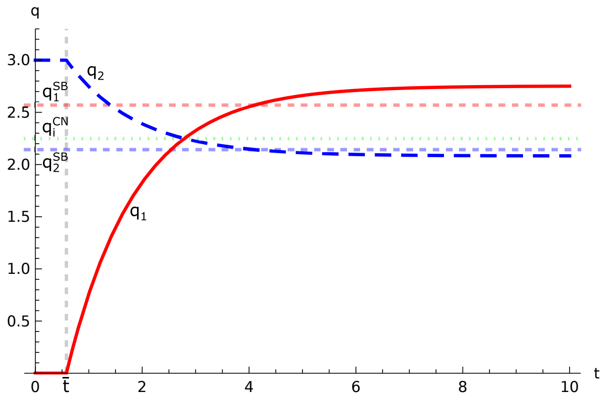
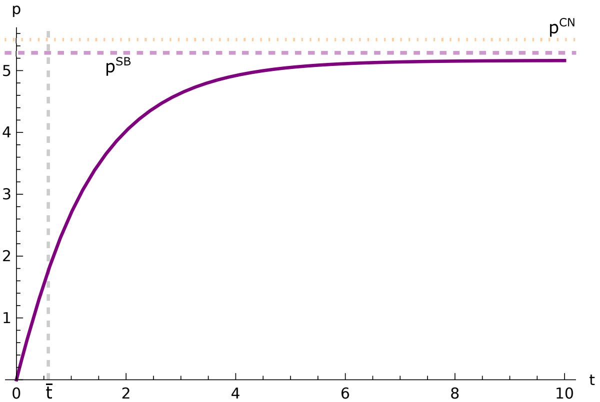
- with p given by the sticky prices equation.
- The unique steady state is stable.
- The open loop and feedback solutions are equivalent, unlike in the analogous Cournot oligopoly model.

The equilibrium production



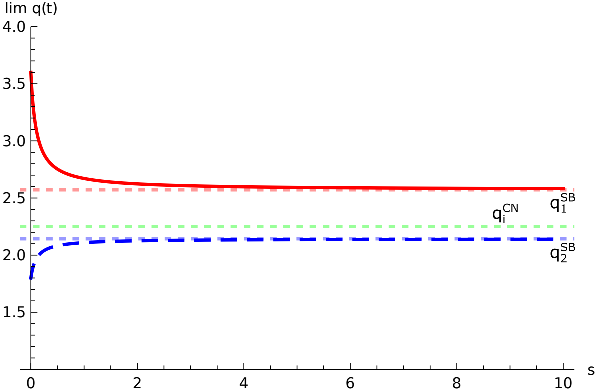
Productions of both players at the strategy maximizing the Leader's payoff compared to static Cournot and Stackelberg equilibrium levels. Solution for model parameters $A = 10$, $c = 1$, $r = 0.15$, $s = 0.5$.

The equilibrium price



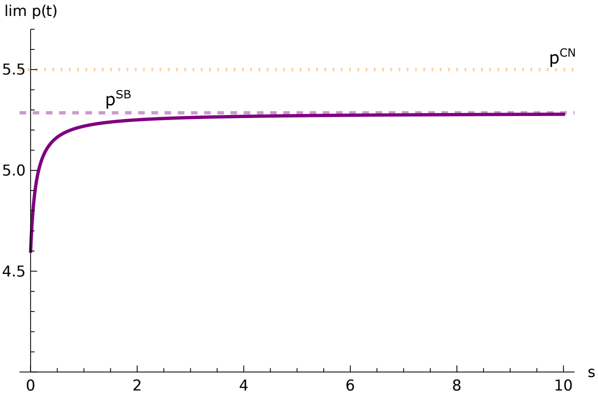
Price at the strategy maximizing the Leader's payoff compared to static Cournot and Stackelberg equilibrium levels. Solution for model parameters $A = 10$, $c = 1$, $r = 0.15$, $s = 0.5$.

Asymptotic behaviour of productions



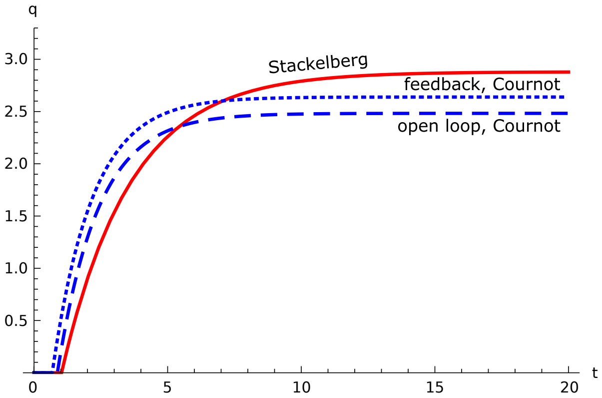
Asymptotic productions of both players as a function of adjustment speed s, for parameters $A = 10$, $c = 1$, $r = 0.15$.

Asymptotic behaviour of price



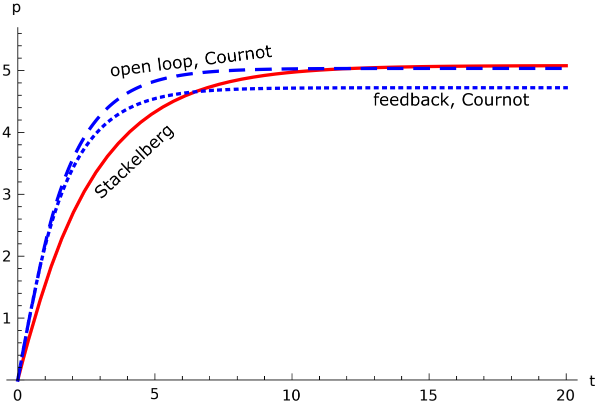
Asymptotic price level as a function of adjustment speed s, for parameters $A = 10$, $c = 1$, $r = 0.15$.

Comparison of Stackelberg and Cournot duopoly—leader's production



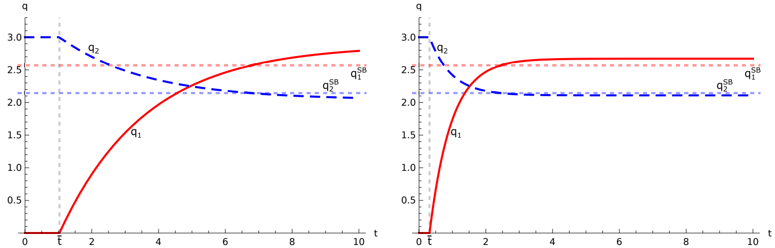
The Stackelberg leader optimal production compared with open loop and feedback equilibrium production of each player in the duopoly model for parameters $A = 10$, $c = 1$, $r = 0.15$, $s = 0.25$.

Comparison of Stackelberg and Cournot duopoly—price



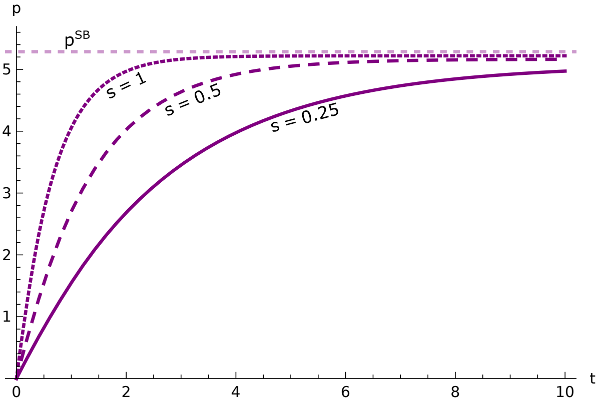
The Stackelberg price compared with the open loop and feedback equilibrium price in the duopoly model for parameters $A = 10$, $c = 1$, $r = 0.15$, $s = 0.25$.

Productions as the speed of adjustment increases



Equilibrium productions for model parameters $A = 10$, $c = 1$, $r = 0.15$.
From the left: $s = 0.25$, $s = 1$.

Price as the speed of adjustment increases



Equilibrium price for model as a function of speed of adjustment.
For model parameters $A = 10$, $c = 1$, $r = 0.15$.

Conclusions

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Danke für Ihre Aufmerksamkeit!